

End-to-End Learning-Based Feedforward Control of a Nonlinear Piezoelectric Actuator

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Abstract

Semiconductor metrology and inspection rely critically on high-precision piezoelectric actuators. This paper presents a learning-based feedforward controller for improving the dynamic tracking performance of piezoelectric actuators in such systems. Unlike conventional operator-based approaches, the proposed method is end-to-end, fully data-driven, with very low computational cost. We introduce Play Recurrent Neural Network Hammerstein (PRNN-H), a modular architecture that combines a parallel Multilayer Perceptron (MLP) for general dynamics with a recurrent hysteresis module, enabling end-to-end training. To enable rapid hardware-in-the-loop evaluation without dedicated real-time equipment, we design a pseudo-real-time dual-layer dual-core controller, in which a microcontroller (MCU) runs fast feedback–feedforward loop while the host PC computes updated feedforward commands via the learned inverse model at low frequency. Real-world experiments on a Physik Instrumente P-621 nanopositioning stage achieve a mean absolute error of 0.13 % of full-scale range, representing a 58 % improvement over the ablation baseline (MCU feedback only) and over 70 % improvement over the built-in E-709 feedback controller.

Piezoelectric Actuator; Feedforward Control; Hysteresis; Recurrent Neural Network; Hammerstein Model; Dynamic Tracking; Precision Motion

1. Introduction

Since 1965, Moore's Law has predicted that transistor density doubles approximately every two years, a pace that broadly held for decades [1,2] but has now confronted fundamental physical limits [3], pushing the industry toward application-driven solutions such as AI accelerators, advanced packaging, and heterogeneous integration [4].

Within this context, metrology and inspection remain among the most essential steps in semiconductor manufacturing, involving 400 to 600 process steps over cycles of one to two months [5]. The global metrology and inspection equipment market was valued at USD 8.98 billion in 2024 and is projected to reach USD 16.21 billion by 2033 [6]. From the IRDS, overlay metrology uncertainty for DRAM must decrease from 0.7 nm to 0.3 nm by 2027 [7,8], demanding sub-nanometer error budgets from the underlying motion systems. These systems combine motorized stages for coarse wafer transport with piezoelectric actuators for precise adjustment and autofocus.

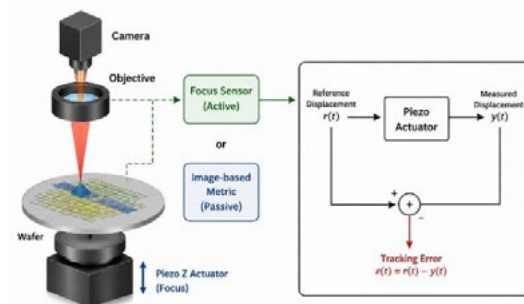


Figure 1. Focus tracking in semiconductor metrology and inspection.

Two autofocus strategies are used in practice as shown in Figure 1. Active autofocus uses an external sensor to continuously command the actuator to the correct focal plane [9]. It is fast but limited in accuracy by sensor noise. Passive autofocus determines focus from acquired images across multiple stepped positions [9], which is more accurate but inherently slower. This paper addresses the dynamic tracking problem underlying active autofocus, which is the key performance bottleneck in current tools.

Piezoelectric actuators present two characteristic modeling challenges: hysteresis, a path-dependent

nonlinearity in which displacement depends on input history and direction [10,11], and creep, a slow output drift under constant input caused by ferroelectric domain relaxation [10,12]. Both effects shift with temperature, humidity, and aging, making a single static model insufficient [10,13]. Conventional operator-based approaches such as the Prandtl-Ishlinskii model represent hysteresis as a superposition of play operators [14] and can serve as invertible feedforward compensators [14,15], but they require large operator banks and dedicated calibration, and must be cascaded with a separately identified linear dynamic element in a Hammerstein structure [16,17]. The identification process is complicated by the inaccessibility of the intermediate signal between the two blocks [16].

Learning-based methods explored in our prior work include ARX models, which are computationally efficient but linear [18,19]; a parallel MLP, which captures general nonlinearities but is not designed for path-dependent behavior [18]; and the Hysteretic Recurrent Neural Network (HRNN), which achieved the best accuracy but requires approximately 15.8 ms per simulation step, precluding real-time deployment [20].

In this paper, we introduce the Play Recurrent Neural Network Hammerstein (PRNN-H), a novel modular architecture for end-to-end data-driven piezoelectric modeling. Using a recurrent neuron formulation that encodes the play operator [24] and composing it with a parallel MLP in a Hammerstein-type structure [25], the model achieves accurate tracking of unseen trajectories with per-step inference well below 1 ms, representing approximately a 19-fold reduction in computation time relative to the HRNN. The modular design also permits adaptation to other dynamical systems with a dominant hysteresis physics.

Building on this model, we demonstrate four experimental configurations on a Physik Instrumente (PI) P-621 nanopositioning stage with real hardware. First, the forward model achieves a simulation mean absolute error (MAE) of 0.68 % of full-scale range (fsr) at 0.83 ms per step. Second, the inverse model achieves 0.71 % MAE at 0.90 ms per step. Third, applied offline as a trajectory shaper in open-loop mode, the inverse model reduces tracking error from 6.38 % to 1.63 % fsr. Fourth, integrated into a pseudo-real-time dual-layer dual-core controller, the full system achieves 0.13 % MAE fsr, a 58 % improvement

over the MCU-only ablation baseline and over 70 % improvement over the built-in E-709 controller.

The contributions of this paper are threefold:

- A PRNN-Hammerstein architecture for end-to-end learning of nonlinear piezoelectric actuator dynamics, with low computational cost, modular structure, and demonstrated generalization to unseen trajectory types.
- A pseudo-real-time dual-layer dual-core controller framework providing an accessible and low-cost test bench for advanced control algorithm development and validation, without dedicated real-time hardware.
- Comprehensive real-world evaluation covering forward modeling, inverse modeling, open-loop feedforward shaping, and pseudo-real-time closed-loop control, with transparent reporting of all numerical baselines.

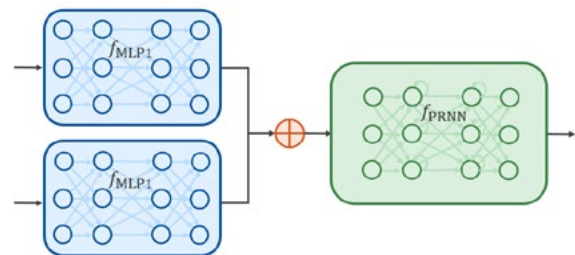


Figure 2. PRNN-H model.

2. Methods

2.1 Problem Formulation

Learning the behavior of a piezoelectric actuator from data can be cast as two supervised learning problems.

- The forward modeling problem: given the history of actuator inputs, predict the output displacement in open-loop simulation without access to the real-time measured signal.
- The inverse modeling problem: given a desired output trajectory, estimate the input voltage sequence that would produce it.

The resulting learned functions are the forward model and inverse model.

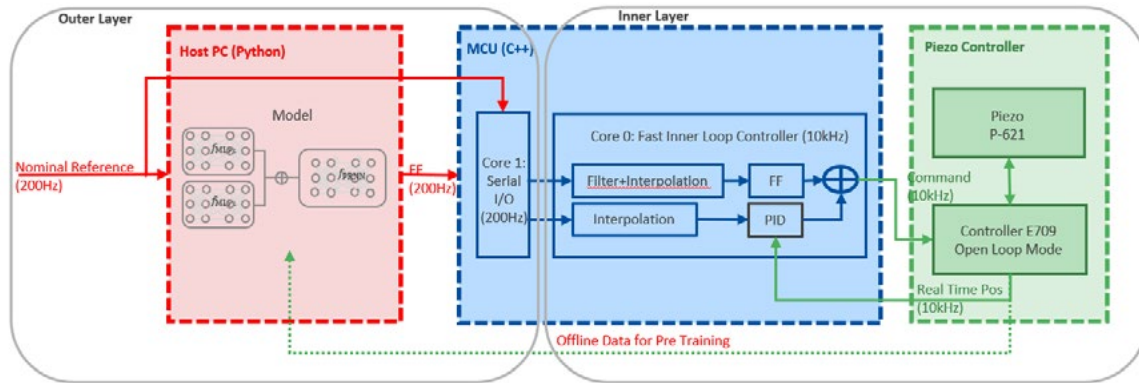


Figure 3. Pseudo-real-time dual-layer control architecture.

2.2. Play Recurrent Neural Network Hammerstein (PRNN-H) Model

The proposed model, as shown in Figure 2, adopts a Hammerstein block structure in which a general-dynamics module and a hysteresis module are cascaded in series and trained end-to-end from input-output data, without requiring access to any intermediate signal. The hysteresis module is based on a recurrent neural network formulation that encodes path-dependent memory, and the dynamics module is based on a parallel two-path structure that separates input-side and output-history processing. Both the forward model and the inverse model share the same architecture, differing only in the roles assigned to inputs and outputs. The full model is trained by backpropagation through time using a mean-squared-error loss.

The proposed model requires sub-millisecond per-step inference on a standard host PC, making it compatible with embedded and near-real-time deployment.

2.3. Dual-Layer Controller

The controller combines a learned feedforward interpolated and filtered before being applied.

The experimental platform is a Physik Instrumente P-621 PIHera Z-stage driven by a PI E-709 digital controller. A dual-core microcontroller bridges the host PC and E-709: Core 0 runs the fast inner feedforward loop via SPI, while Core 1 handles slow serial communication with the host. The host PC runs the inverse model at low frequency and transmits updated feedforward commands to Core 1. Core 0 runs a PID feedback term for stabilization and disturbance rejection. This architecture is designated pseudo-real-time, as the host operates under a

general-purpose OS rather than a deterministic real-time environment.

3. Experimental Design

Both models are trained offline using data collected from the P-621 stage [26] with the E-709 [27] in open-loop mode.

A total of 50 multisine trajectories is generated [23]. The dataset is split 80/20 by trajectory into training and validation sets. The test trajectory, a 2 Hz full-range triangular wave, is held out entirely and never used during training or validation, representing a qualitatively different waveform class from the multisine training data.

Tracking performance is quantified by the percentage mean absolute error normalized to the full output range. Lower MAE is better.

Baselines are: (1) the E-709 built-in servo controller, and (2) an ablation replacing the learned feedforward with a fixed-gain term.

Four experiments are conducted: (1) offline forward simulation, (2) offline inverse modeling, (3) open-loop feedforward shaping via E-709 wave generator, and (4) pseudo-real-time closed-loop feedforward-feedback control.

4. Results and Discussion

4.1 Experiment 1: Forward Simulation

Figure 4(a) shows the simulation results on the test trajectory (2 Hz full-range triangular wave). The top panel shows predicted (dashed) and recorded (solid) displacements over time; the middle panel shows the hysteresis loop (input voltage vs. displacement); the bottom panel shows the per-step error as a

percentage of full-scale range. The PRNN-H forward model achieves MAE = 0.68 % fsr.

The time-domain traces show tight alignment. In the hysteresis loop, the model accurately reproduces both the ascending and descending branches, including the characteristic asymmetry between them.

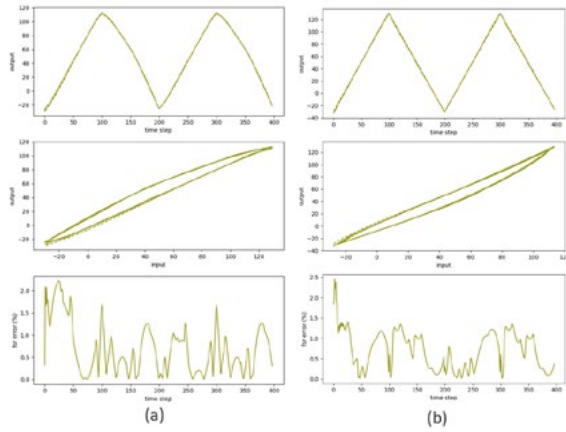


Figure 4. (a) Model simulation (forward modeling) result; (b) Inverse modeling result.

4.2 Experiment 2: Inverse Modeling

Figure 4(b) shows the inverse modeling results. The top panel displays estimated (dashed) and recorded (solid) input voltage sequences; the middle panel shows the estimated versus recorded input-output map; the bottom panel shows the per-step voltage estimation error. The inverse model achieves MAE = 0.71 % fsr, closely matching the forward model.

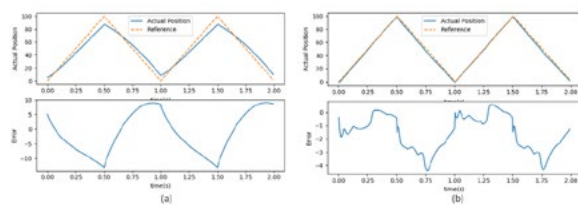


Figure 5. (a) E-709 open loop tracking performance; (b) PRNN-H feedforward shaper tracking performance.

4.3 Experiment 3: Open-Loop Feedforward Shaping

Figures 5(a) and 5(b) compare the PRNN-H feedforward shaper against the E-709 in open-loop mode. Each panel shows the actual position and reference (top) and the tracking error in micrometers (bottom).

In E-709 open-loop mode, see Figure 5(a), the actuator exhibits pronounced hysteresis. The position lags the reference substantially during both ascending and descending phases, with peak errors reaching

approximately $\pm 12 \mu\text{m}$ against the full $100 \mu\text{m}$ range, corresponding to MAE = 6.38 % fsr. This confirms that the raw actuator, driven without any model-based compensation, has significant nonlinear distortion.

With the PRNN-H feedforward shaper, as shown in Figure 5(b), the shaped voltage command pre-compensates for the dominant hysteresis, and the tracking error is substantially reduced. MAE falls from 6.38 % to 1.63 % fsr, a reduction of approximately 74 %.

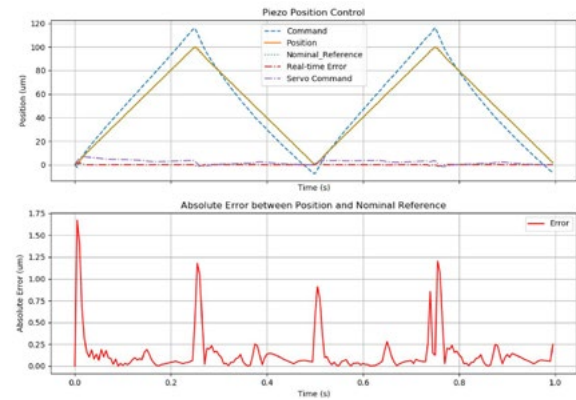


Figure 6. Tracking performance of our controller with feedforward-feedback control

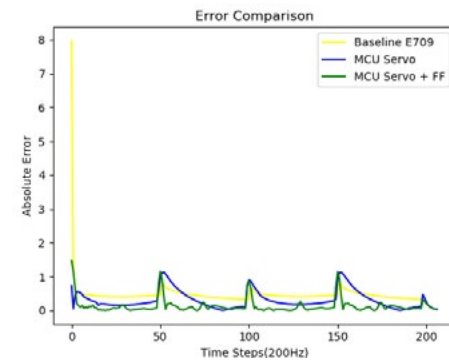


Figure 7. Error comparison over baseline

4.4 Experiment 4: Pseudo-Real-Time Feedforward-Feedback Control

Figure 6 shows the full pseudo-real-time controller result. The top panel displays the actual position (orange) closely tracking the nominal reference (green dotted), with the shaped feedforward command (blue dashed) and PID servo command (magenta) shown alongside. The bottom panel shows absolute error in micrometers. The full controller achieves MAE = 0.13 % fsr.

Figure 7 compares all three conditions directly. The E-709 baseline (yellow) achieves MAE = 0.49 % fsr. The ablation, MCU servo only (blue), achieves 0.31 % fsr.

The proposed full controller, MCU servo with PRNN-H feedforward (green), achieves 0.13 % fsr. The improvement of the full controller over the ablation is 58 %, isolating the contribution of the learned feedforward. The improvement over the E-709 baseline is 73 %. All quantitative results are summarized in Table 1.

Table 1. Tracking Errors

	Configuration	MAE fsr
Modeling	Forward simulation	0.68 %
	Inverse modeling	0.71 %
Control	Open-loop FF shaping	1.63 %
	E-709 open loop (baseline)	6.38 %
	E-709 Servo (baseline)	0.49 %
	MCU Servo only (ablation)	0.31 %
	MCU Servo + PRNN-H FF (ours)	0.13 %

Conclusion

This paper presented a learning-based feedforward controller for dynamic tracking of a nonlinear piezoelectric actuator, motivated by the precision requirements of semiconductor metrology and inspection. The proposed PRNN-Hammerstein model integrates a recurrent play-operator network for hysteresis compensation with a parallel MLP for general dynamics, enabling end-to-end training from raw input-output data without access to any intermediate physical variable. The model achieves sub-millisecond inference, making it compatible with embedded real-time deployment.

To enable practical hardware-in-the-loop evaluation without dedicated real-time equipment, we introduced a pseudo-real-time dual-layer dual-core controller using a microcontroller as a real-time intermediary between the host PC and the PI E-709 driver. This architecture decouples the high-rate

feedback loop from the computationally heavier feedforward inference, with the feedforward command interpolated to 10 kHz inside the MCU.

Real-world experiments on the PI P-621 nanopositioning stage demonstrated progressive improvements across four configurations: forward simulation (MAE 0.68 % fsr), inverse modeling (0.71 % fsr), open-loop feedforward shaping (1.63 % fsr, a 74 % reduction from the 6.38% open-loop baseline), and pseudo-real-time closed-loop control (0.13 % fsr, 58 % better than the ablation and 73 % better than the E-709 baseline).

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